

Fundamentals

- ★ Particle Size Distribution
- ★ Specific Surface Area
- ★ Distribution Function
- ★ Grade Efficiency Curve

HST: The Majestic
Sombrero Galaxy



Fundamentals of Particle Size Analysis

- ★ Introduction
- ★ Particle size distribution
 - ☆ Physical particle properties
 - ☆ Types of quantity
 - ☆ Measures of quantity
 - ☆ Equivalent diameters
 - ☆ PSA methods
- ★ Graphic presentation of PSD
 - ☆ Cumulative distribution $Q_r(x)$
 - ☆ Density (differential) distribution $q_r(x)$
- ★ Specific surface area
 - ☆ Definition
 - ☆ Evaluation from PSD
 - ☆ Blaine principle
- ★ Distribution functions
 - ☆ RRSB distribution
 - ☆ RRSB distribution grid
- ★ Grade efficiency curve
 - ☆ Definition
 - ☆ Tromp curve
 - ☆ Characterisation of a separation
- ★ References



Introduction [3]

- ★ Particle size analysis comprises the measurement and the quantitative description of physical properties of particulate matter
- ★ Particulate or disperse systems consist of a finely divided particulate phase in a continuous carrier or dispersion medium
- ★ Particulate phase and dispersion medium may be solid, liquid or gaseous
- ★ Particle size analysis predominantly investigates such systems as suspensions, emulsions, aerosols (sprays) and aero-dispersions (dry powders)
- ★ In PSA every possible physical principle has been used to determine the particle size distribution
 - ☆ Only very few have reached an important position in industrial application, e.g.:
 - ↳ Sieving
 - ↳ Sedimentation
 - ↳ Laser diffraction



Particle Size Distribution [3]

Individual elements of disperse matter form a distribution. It is elements can be classified according the their size.

The distribution is characterised by a quantity, e.g. the number or the mass of particles present in individual size classes.

Physical particle properties used in PSA are:

- ★ Geometric properties
 - ☆ Linear dimension
 - ☆ Surface area, projected area
 - ☆ Volume
- ★ Mass
- ★ Settling velocity
- ★ Distortion of electrical, optical, acoustic field



The types of quantity used in PSA are defined as follows:

Quantity type	Index r
Number	0
Length	1
Area, surface	2
Volume	3
mass	3 (if $\rho_s = \text{const.}$)

Measures of quantity used in PSA are:

- ★ Cumulative distribution Q_r
- ★ Density or differential distribution q_r

Equivalent diameters are diameters of spheres that yield the same value of a certain physical property when analysed under the same conditions as the irregularly shaped particles



PSA methods related to physical properties and quantity types, e.g.:

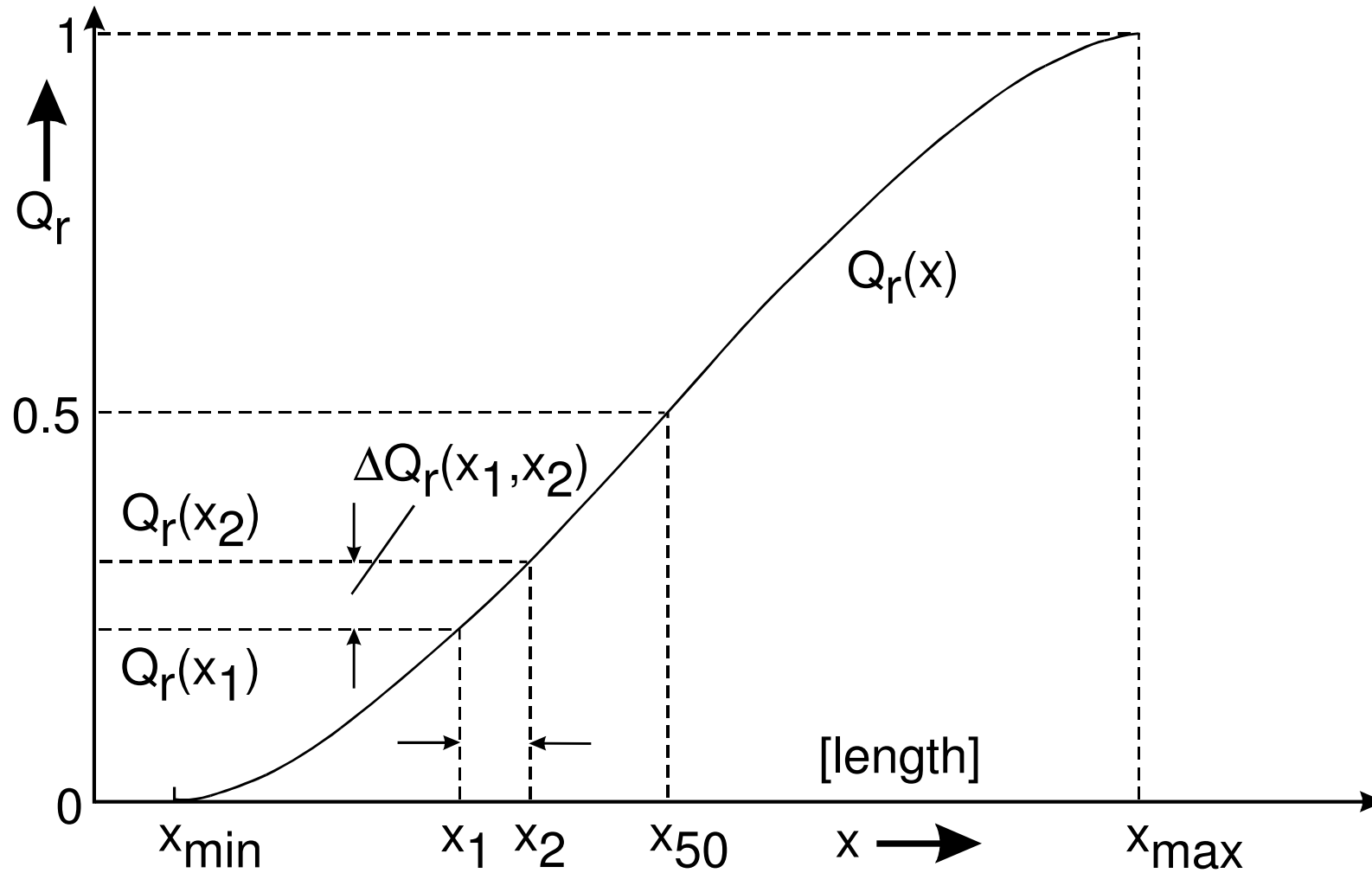
Method	Particle property	Quantity type
Sieve analysis	linear dimension	mass
Sedimentation	settling velocity	mass
Coulter counter	volume	number
Laser diffraction	distortion of optical field	area

Graphic Presentation of PSD [4]

In a two dimensional graphic presentation of PSA data, the independent variable, plotted on the abscissa, describes the physical property chosen to characterise the size of the particles.

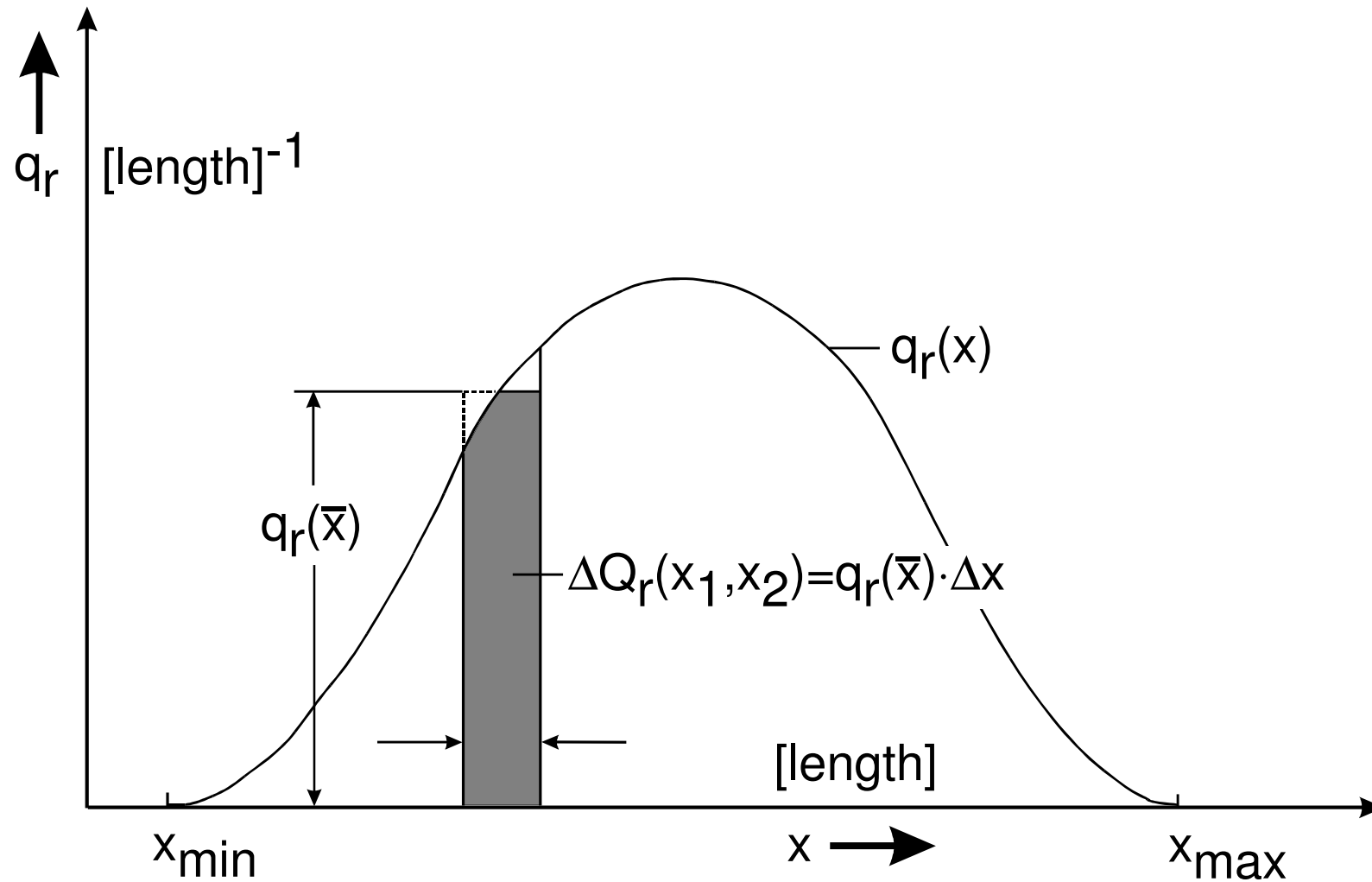
The dependent variable characterises type of quantity and measure of quantity and is plotted on the ordinate (refer to ISO 9276-1).





Cumulative distribution





Density of differential distribution



The shaded area in the density or differential distribution curve

$$dQ_r(x) = q_r(x) dx$$

represents the quantity of particles $dQ_r(x)$ between x and $x + \Delta x$.

The area below the density distribution curve $q_r(x)$ always equals ONE (condition of normalisation) !

$$\int_{x_{\min}}^{x_{\max}} q_r(x) dx = Q_r(x_{\max}) - Q_r(x_{\min}) = 1$$



Specific Surface Area [5]

Definition:

Volume related specific surface

$$S_V = S/V \quad [\text{m}^2 / \text{cm}^3]$$

Mass related specific surface

$$S_m = S/m \quad [\text{cm}^2 / \text{g}]$$

$$S_m = S_V / \rho_s$$

$$\rho_s = \text{density of solid}$$



Evaluation from Particle Size Distribution

Definition of momentum:

$$M_{k,r} = \int_{x_{\min}}^{x_{\max}} x^k \cdot q_r(x) dx$$

- k = exponent k of equivalent diameter x
 r = quantity measure of q
 k = ... -3, -2, -1, 0, 1, 2, 3 ...
 r = 0, 1, 2, 3

$$S_V = 6 \cdot f \cdot \frac{M_{2.0}}{M_{3.0}} = 6 \cdot f \cdot M_{-1.3}$$

$$S_V = 6 \cdot f \cdot \int_{x_{\min}}^{x_{\max}} \frac{1}{x} \cdot q_3(x) dx$$

f = Heywood shape factor



Blaine Principle: (DIN 66127)

$$S_V^2 = K \cdot \frac{\varepsilon^3 \tau}{(1-\varepsilon)^2 \eta} \quad [\text{m}^2 / \text{cm}^3]$$

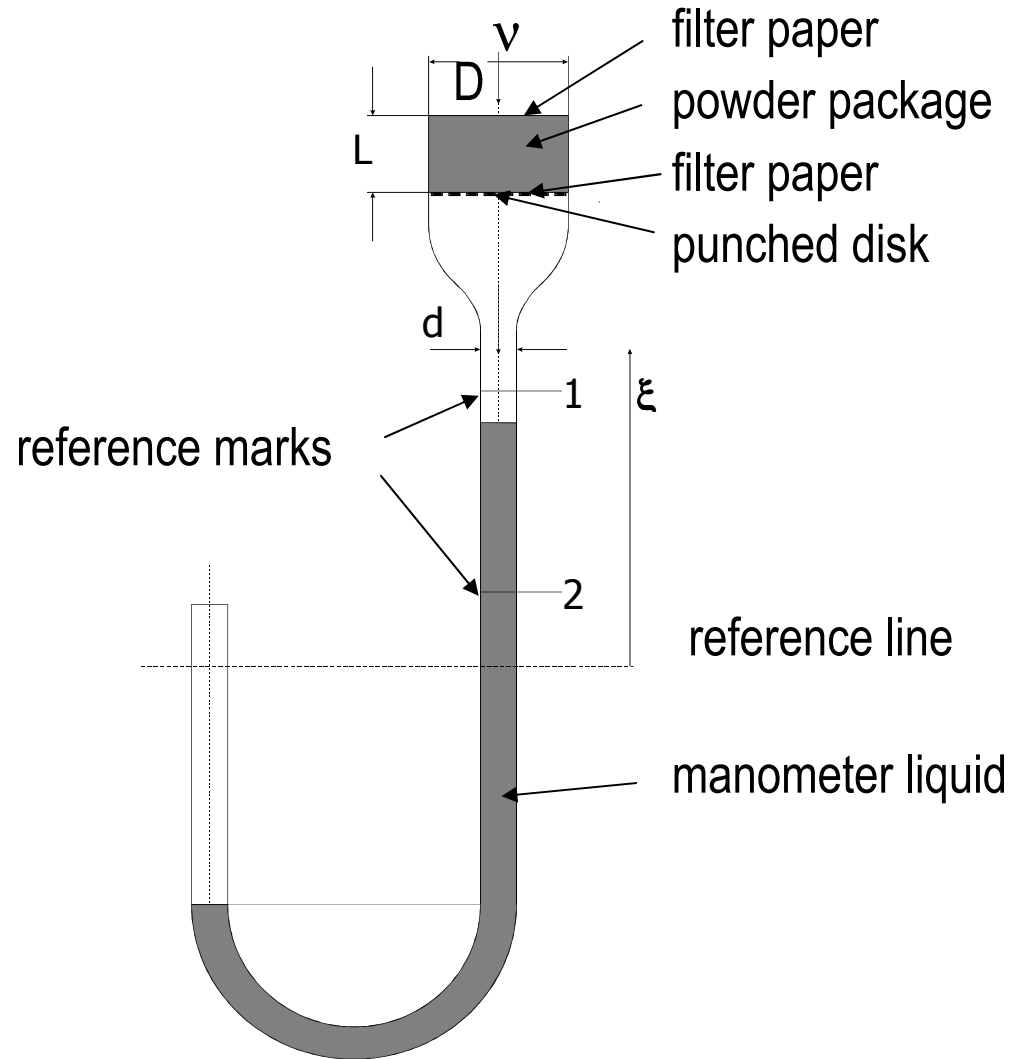
- S_V = Volume related specific surface
- K = Instrument constant
- ε = void ratio, permeability, porosity
- τ = time
- η = dynamic viscosity

$$K = \frac{2}{k} \cdot \frac{g}{L} \cdot \frac{\rho_M}{\ln(\xi_1 / \xi_2)} \cdot \left(\frac{D}{d} \right)$$

- k = Kozeny constant
- g = Gravity constant
- L = Height of bulk package
- ρ_M = Density of manometer liquid
- D = Diameter of package
- d = Diameter of capillary
- $\ln(\xi_1 / \xi_2)$ = Height of manometer liquid between position 1 and 2



Principle of Blaine instrument



Distribution Functions

RRSB distribution (DIN 66145)

(Rosin, Rammler, Sperling, Bennet)

$$R(x) = 1 - Q_3(x) = \exp \left(- \left(\frac{x}{x'} \right)^n \right)$$

Two parameter function:

x' = Position parameter

n = Scatter parameter

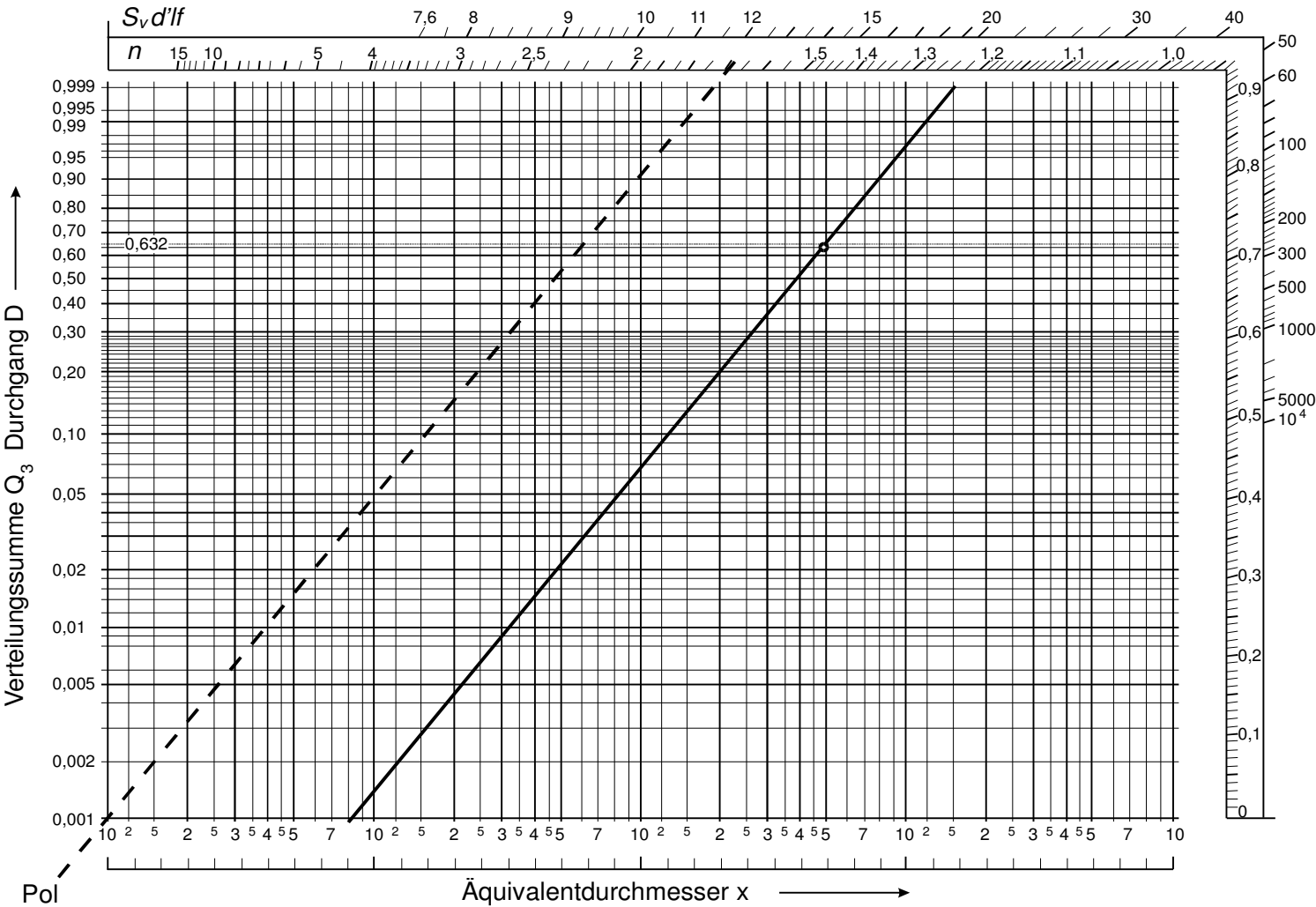
$$\begin{aligned} x' : R_3(x') &= e^{-1} = 0,368 \\ &Q_3(x') = 0,632 \end{aligned}$$

n : Slope of straight line in RRSB grid



RRSB distribution grid

refer to DIN 66145



Grade Efficiency Curve T(x) [3]

Definition

The grade efficiency curve T(x) (Tromp Curve) describes the efficiency of a classification or a separation process.

T(x) represents the ratio of the relative amount of material of certain size present in the coarse material (oversize, reject) ($C \cdot q_C(x) dx$), to the relative amount of the same size initially present in the feed material ($q_F(x) dx$).

$$T(x) = \frac{C \cdot q_C(x)}{q_F(x)}$$

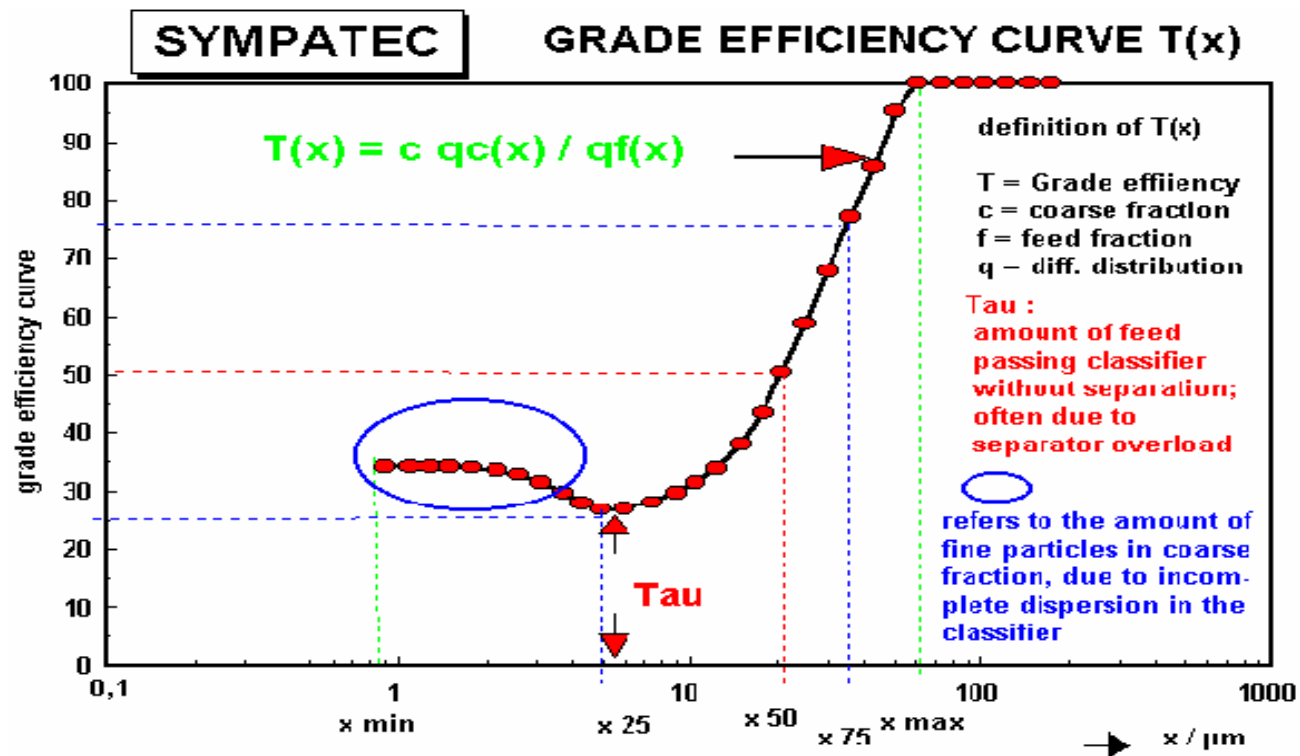
C = relative amount of coarse material

$$C = \frac{M_C}{M_F} = \frac{M'_C}{M'_F}$$

$M_F, M'_F =$ amount mass flow rate of feed material

$M_C, M'_C =$ amount mass flow rate of coarse material



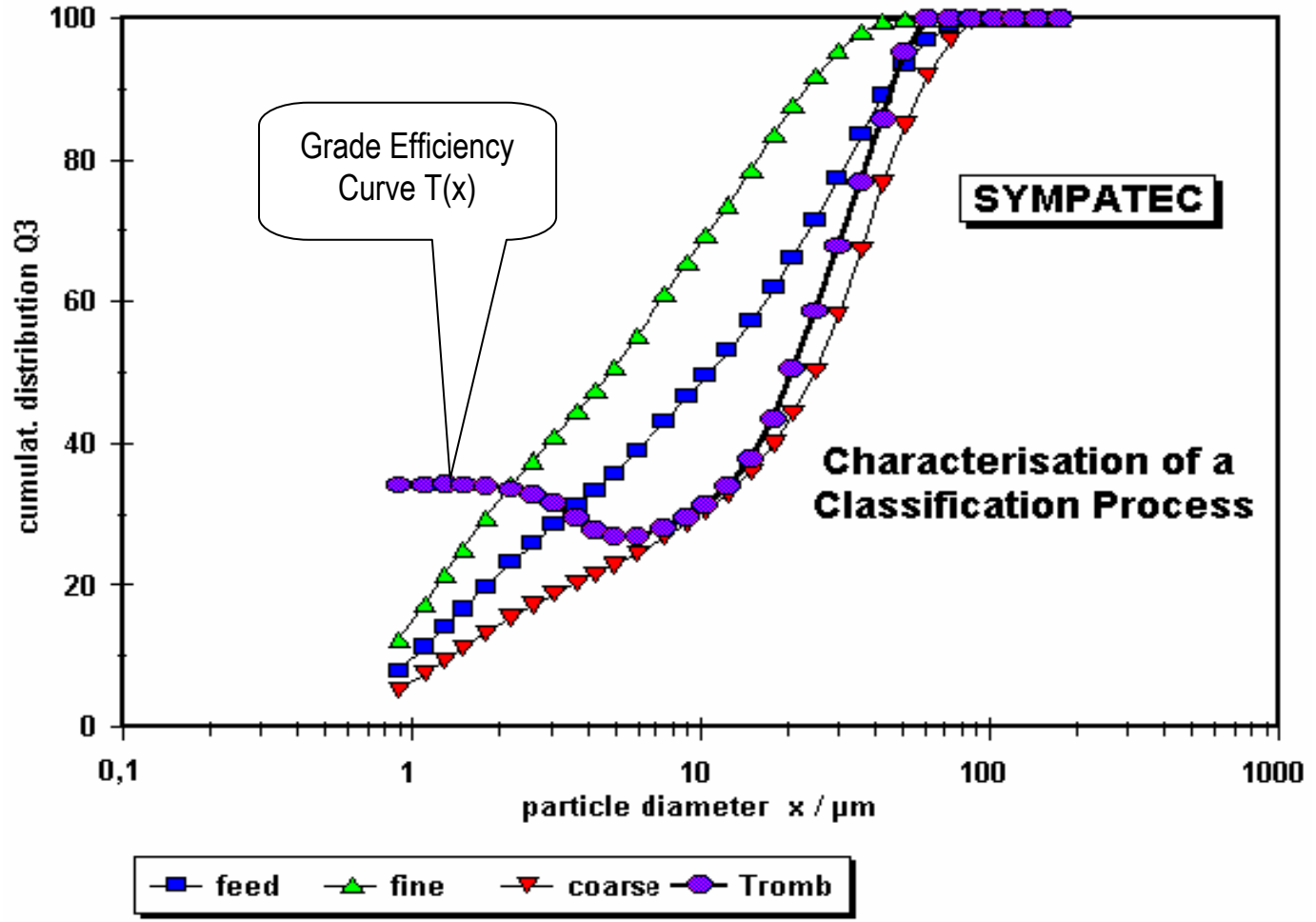


Measures for sharpness of cut

Ecat Terra: $E_T = (x_{75} - x_{25})/2$

Imperfection: $I = (x_{75} - x_{25})/2 \cdot x_{50}$





References

- [1] Particle Technology
(Mechanische Verfahrenstechnik)
Hans Rumpf
CHAPMAN and HALL, 1990
ISBN 0-412-35230-3
- [2] Particle Size Measurement
Terence Allen
CHAPMANN and HALL, 4th edition
ISBN 0-412-35070-X
- [3] Representation and Evaluation
of Particle Size Analysis Data
Kurt Leschonski
Particle Characterisation 1
(1984), 89 – 95
- [4] Representation of Results of Particle
Size Analysis
ISO 9276-1: Graphical representation
ISO 9276-2: calculation of average
particle size
- [5] Darstellung und Auswertung von
Partikelgrößenmessungen
DIN 66141 – 66145
- [6] Perry Chemical Engineer's
Handbook; 8th ed.
Robert H. Perry
Chapter Particle Sizing
Dr. Wolfgang Witt

